Write the formal definition of "onto". **SCORE: _____ / 8 PTS**

GIVEN SETS A, B AND FUNCTION $f: A \to B$,
f IS ONTO IFF FOR ALL $y \in B$
THERE IS AN $x \in A$
SUCH THAT $f(x) = y$

Write the formal definition of "transitive". **SCORE: _____ / 8 PTS**

A RELATION R ON SET A IS TRANSITIVE
IFF FOR ALL $x, y, z \in A$,
IF xRy AND yRz
THEN xRz

Write a formal proof that $A = \{x \in R \mid 0 < x < 1\}$ and $B = \{x \in R \mid 2 < x < 5\}$ have the same cardinality. **SCORE: _____ / 30 PTS**
NOTE: The simpler your one-to-one correspondence is, the simpler your proof will be.

LET $f(x) = 3x + 2$ $\leftarrow$ ALSO COULD USE $f(x) = 5 - 3x$

LET $x, z \in A$ SUCH THAT $f(x) = f(z)$
SO $3x + 2 = 3z + 2$
$3x = 3z$
$x = z$
SO f IS 1-1

SEE 7.4 #11

LET $y \in B$
SO $2 < y < 5$
CONSIDER $x = \frac{y-2}{3}$
$f(x) = 3\left(\frac{y-2}{3}\right) + 2 = y - 2 + 2 = y$
AND SINCE $2 < y < 5$
$0 < y - 2 < 3$
$0 < \frac{y-2}{3} < 1$
SO $0 < x < 1$
SO $x \in A$
SO f IS ONTO

SINCE $f: A \to B$ IS A ONE-TO-ONE CORRESPONDENCE, THEREFORE A, B HAVE THE SAME CARDINALITY

Let $A = Z^+ \times Z^+$ and let R be the relation on A defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = c + b$. **SCORE: _____ / 35 PTS**

[a] **<u>Write a formal proof</u>** that R is transitive.

LET $(a,b),(c,d),(e,f) \in A$ SUCH THAT $(a,b)R(c,d)$ AND $(c,d)R(e,f)$

SO $a+d=c+b$ AND $c+f=e+d$

SO $a+f=(c+b-d)+f$
$=(c+f)+b-d$
$=e+d+b-d$
$=e+b$

SO $(a,b)R(e,f)$

SO R IS TRANSITIVE

SEE 8.3 #38c

[b] In fact, R is an equivalence relation. **<u>(NOTE: You do NOT need to prove R is an equivalence relation.)</u>**
Find 5 elements of $[(3, 1)]$.

$(4,2), (5,3), (6,4), (7,5), (8,6) \ldots$

$(a,b)R(3,1)$ IFF $a+1=3+b$
$a=b+2$

SEE 8.3 #38e

How many 5 card poker hands contain one pair (ie. two cards of one value, and three other cards all of different values from each other as well as the pair) ? Remember that a standard deck has 4 suits, each with 13 values, for a total of 52 cards. **SCORE: _____ / 15 PTS**

1 PICK 1 VALUE FOR PAIR $C(13,1)=13$

2 PICK 2 SUITS FOR PAIR $C(4,2)=6$

3 PICK 3 VALUES FOR SINGLES (NOT 1) $C(12,3)=\frac{\cancel{12}^{2}\cdot 11\cdot 10}{\cancel{3\cdot 2}\cdot 1}=220$

4 PICK 1 SUIT FOR LOWEST SINGLE $C(4,1)=4$

5 MIDDLE $C(4,1)=4$

6 HIGHEST $C(4,1)=4$

SEE 9.5 #11h

TOTAL $=13\cdot 6\cdot 220\cdot 4^3$

Let R be an equivalence relation on set A. **Write a formal proof** for the following statement. **SCORE: _____ / 30 PTS**

Use the definitions in sections 8.2 and 8.3 but do NOT use any of the lemmas, theorems or homework exercises as justification.

For all $a, b \in A$, if $a \in [b]$, then $[a] = [b]$.

SEE 8.3 # 35

LET $a, b \in A$ SUCH THAT $a \in [b]$

SO aRb BY DEF'N OF [ ]

LET $x \in [a]$

SO xRa BY DEF'N OF [ ]

SINCE xRa AND aRb,

THEREFORE xRb BY TRANSITIVITY

SO $x \in [b]$ BY DEF'N OF [ ]

SO $[a] \subseteq [b]$ BY DEF'N OF $\subseteq$

LET $x \in [b]$

SO xRb BY DEF'N OF [ ]

SINCE aRb,

THEREFORE bRa BY SYMMETRY

SINCE xRb AND bRa,

THEREFORE xRa BY TRANSITIVITY

SO $x \in [a]$ BY DEF'N OF [ ]

SO $[b] \subseteq [a]$ BY DEF'N OF $\subseteq$

SO $[a] = [b]$ BY DEF'N OF =

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5 couples attend the theater together. **SCORE: _____ / 24 PTS**

[a] If each couple wants to sit together (ie. next to each other), how many ways can the 10 people be seated together in a row ?

① ARRANGE 5 COUPLES ("GLUED" TOGETHER) $5!$

② ARRANGE COUPLE 1 (AB OR BA) $2!$

③ 2 $2!$

④ 3 $2!$

⑤ 4 $2!$

⑥ 5 $2!$

TOTAL = $2^5 \cdot 5!$

[b] Pat and Chris Kim just had an argument. How many ways can the 10 people be seated together in a row so that Pat and Chris do not sit together ? **NOTE: This question is NOT related to [a].**

PAT + CHRIS SIT TOGETHER

① ARRANGE PAT-CHRIS AND 8 OTHER PEOPLE $9!$

② ARRANGE PAT-CHRIS (PC OR CP) $2!$

SUBTOTAL = $2 \cdot 9!$

PAT + CHRIS DO NOT SIT TOGETHER

TOTAL = $10! - 2 \cdot 9!$

[c] In addition, Reese and Kyle Hunter are going through a trial separation. How many ways can the 10 people be seated together in a row so that Pat and Chris do not sit together, and Reese and Kyle do not sit together ? **NOTE: This question is a continuation of [b].**

PAT + CHRIS SIT TOGETHER SUBTOTAL = $2 \cdot 9!$

REESE + KYLE SIT TOGETHER SUBTOTAL = $2 \cdot 9!$

BOTH COUPLES SIT TOGETHER SUBTOTAL = $2^2 \cdot 8!$

EITHER COUPLE SIT TOGETHER SUBTOTAL = $2 \cdot 9! + 2 \cdot 9! - 2^2 \cdot 8!$

NEITHER COUPLE SIT TOGETHER TOTAL = $10! - 2 \cdot 9! - 2 \cdot 9! + 2^2 \cdot 8!$